PCMI topological aspects of quantum codes, problem session #4

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1. **(Correcting the T Injection.)** In the T injection circuit, we prepare an ancilla in the $|T\rangle$ state, and then measure ZZ on the ancilla and target, and finally measure X on the ancilla qubit. Write the post-measurement state of the system after all 4 possible measurement outcomes (00, 01, 10, 11), and determine the corrections needed, ignoring global phase factors.

2. **(Faulty T Injection.)** What happens if instead of $|T\rangle$, the T injection circuit is run with the state $Z|T\rangle$? Consider perfect Clifford operations but nonideal T ancilla state. What is the noise on the data qubit after the injection? What Clifford twirling was applied to the ancilla?

3. **(Level 3 Divisible Implies Triorthogonal.)** Recall the definitions of level-3 divisibility and triorthogonal:

   (a) A vector subspace V of m bits, equipped with an odd integer vector, t, is *level-*$(3,t)$ *divisible* if for all $v \in V$, $v \cdot t \mod 8 = 0$.

   (b) A $(k+s)\times n$ binary matrix is *triorthogonal* if the first (1) k rows have odd weight, (2) the last s rows have even weight, (3) the bitwise AND of every pair of rows has even weight, and (4) the bitwise AND of every triple of rows has even weight.

   Show that a level-3 divisible subspace has a triorthogonal generator matrix, where every row has even weight.

4. **(Level-4 Divisibility.)** Define level-4 divisibility, and show that the $3D$ color code's X stabilizer group is *not* level-4 divisible.